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On the IR/UV mixing and experimental limits on the parameters of canonical noncommutative spacetimes

Giovanni Amelino-Camelia, Gianluca Mandanici and Kensuke Yoshida

Dipart. Fisica, Univ. Roma "La Sapienza" and INFN Sez. Roma 1
P.le Moro 2, 00185 Roma, Italy
E-mail: amelino@roma1.infn.it, gianluca.mandanici@roma1.infn.it, yoshida@roma1.infn.it

Abstract: We investigate some issues that are relevant for the derivation of experimental limits on the parameters of canonical noncommutative spacetimes. By analyzing a simple Wess-Zumino-type model in canonical noncommutative spacetime with soft supersymmetry breaking we explore the implications of ultraviolet supersymmetry on low-energy phenomenology. The fact that new physics in the ultraviolet can modify low-energy predictions affects significantly the derivation of limits on the noncommutativity parameters based on low-energy data. These are, in an appropriate sense here discussed, "conditional limits". We also find that some standard techniques for an effective low-energy description of theories with non-locality at short distance scales are only applicable in a regime where theories in canonical noncommutative spacetime lack any predictivity, because of the strong sensitivity to unknown UV physics. It appears useful to combine high-energy data, from astrophysics, with the more readily available low-energy data.

Keywords: Renormalization Group, Supersymmetry Breaking, Supersymmetry Phenomenology, Non-Commutative Geometry.

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Contents

1. Introduction

Recently, there has been strong interest (see, e.g., refs. [1, 2, 3]) in quantum field theories constructed on canonical[1] noncommutative spacetime:

$$[x_\mu, x_\nu] = i\theta_{\mu\nu}\,. \tag{1.1}$$

This recent interest is mostly due to the possible use of these spacetimes in effective-theory descriptions of string theory in presence of an external background field, in which case $\theta_{\mu\nu}$ reflects the properties of the background. Previously the same algebraic relations provided the basis [7, 8] for an approach to the fundamental description of spacetime physics.

A key characteristic of field theories on canonical spacetimes, which originates from the commutation rules, is nonlocality. At least in the case of space/space noncommutativity ($\theta_{0i} = 0$), to which we limit our analysis for simplicity,[2] this nonlocality is still tractable although it induces a characteristic mixing of the ultraviolet and infrared sectors of the

[1] Interest in the κ-Minkowski [4] Lie-algebra noncommutative spacetime, $[x_m, t] = i\lambda x_m$, $[x_m, x_l] = 0$, has also grown recently, especially because of its possible role in the new relativistic theories with two observer-independent scales [5, 6]. We here focus exclusively on canonical noncommutative spacetimes (1.1).

[2] The case of space/time noncommutativity ($\theta_{0i} \neq 0$) is not necessarily void of interest [9], but it is more delicate, especially in light of possible concerns for unitarity. Since our analysis is not focusing on this point we will simply assume that $\theta_{0i} = 0$.

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theory. This IR/UV mixing has wide implications, including the possible emergence of infrared (zero-momentum) poles in the one-loop two-point functions. In particular one finds a quadratic pole for some integer-spin particles in non-SUSY theories [2], while in SUSY theories the poles, if at all present, are logarithmic [3, 10, 11]. It is noteworthy that these infrared singularities are introduced by loop corrections and originate from the ultraviolet part of the loop integration: at tree level the two-point functions are unmodified, but loop corrections involve the interaction vertices, which are modified already at tree level.

There has been considerable work attempting to set limits on the noncommutativity parameters θ by exploiting the modifications of the interaction vertices [12]–[15] and the modifications of the dressed/full propagators [16]. Most of these analyses rely on our readily available low-energy data. The comparison between theoretical predictions and experimental data is usually done using a standard strategy (the methods of analysis which have served us well in the study of conventional theories in commutative spacetime). We are here mainly interested in understanding whether one should take into account some of the implications of the IR/UV mixing also at the level of the techniques by which one compares theoretical predictions with data. In ref. [16] it was argued that the way in which low-energy data can be used to constrain the noncommutativity parameters is affected by the IR/UV mixing. These limits on the entries of the θ matrix might not have the usual interpretation: they could be seen only as "conditional limits", conditioned by the assumption that no contributions relevant for the analysis are induced by the ultraviolet. The study we report here is relevant for this delicate issue. By analyzing a simple noncommutative Wess-Zumino-type model, with soft supersymmetry breaking, we explore the implications of ultraviolet supersymmetry on low-energy phenomenology. Based on this analysis, and on the intuition it provides about other possible features of ultraviolet physics, we provide a characterization of low-energy limits on the noncommutativity parameters. Our analysis provides additional encouragement for combining, as proposed in ref. [16], high-energy data, from astrophysics, with the more readily available low-energy data.

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2. Preliminaries on the IR/UV mixing

The construction of quantum field theories in canonical noncommutative spacetime is usually obtained by means of the Weyl map that acts between the functions on noncommutative $\mathcal{R}^4$ and the functions on commutative $\mathcal{R}^4$. The Weyl map associates to the product of two functions on noncommutative $\mathcal{R}^4$ the "$\star$" (Moyal) product

$$f(x) \star g(x) := e^{\frac{i}{2}\theta^{\mu\nu}\partial_\mu^y \partial_\nu^z} f(y)g(z)\Big|_{y=z=x}, \tag{2.1}$$

which is noncommutative, but associative. As mentioned, at tree level the $\star$ product induces a modification of the interaction vertices, which acquire characteristic θ- and momentum-dependent phases, while the tree-level propagator is unaffected. At one-loop level the modified vertices generate θ-dependent corrections to the propagator. Let us revisit briefly, in the illustrative example of the "$\lambda\Phi^4$" scalar-boson field theory, the IR/UV mixing that affects these θ-dependent corrections to the propagator. Adopting the standard strategy of distinguishing between planar and nonplanar diagrams [2, 3, 17], one finds a planar tadpole contribution characterized by integrals of the form:

$$\int_0^\Lambda dk \frac{k^3}{k^2+m^2} = \frac{1}{2}\Lambda^2 - \frac{1}{2}m^2 \ln\left(1+\frac{\Lambda^2}{m^2}\right) . \tag{2.2}$$

and a corresponding nonplanar tadpole contribution of the type:

$$\int_0^\Lambda dk \cos\left(\frac{1}{2}k\widetilde{p}\right) \frac{k^3}{k^2+m^2} , \tag{2.3}$$

where we introduced a momentum cutoff Λ and the standard notation $\widetilde{p}_\mu \equiv \theta_{\mu\nu}p_\nu$.

As well known, the integral (2.2) is cut off by Λ while its nonplanar counterpart (2.3), is cut off by the smaller between Λ and $|\widetilde{p}|^{-1}$. In fact, for $\Lambda \ll |\widetilde{p}|^{-1}$ the θ-dependent phase in (2.3) is insignificant, while for $\Lambda \gg |\widetilde{p}|^{-1}$ the integrand of (2.3) oscillates rapidly in the region where the integration momentum k is such that $k \gg |\widetilde{p}|^{-1}$:

$$\int_0^\Lambda dk \cos\left(\frac{1}{2}k\widetilde{p}\right) \frac{k^3}{k^2+m^2} \simeq \frac{1}{2}\left(\frac{2}{|\widetilde{p}|}\right)^2 - \frac{1}{2}m^2 \ln\left(1+\frac{\left(\frac{2}{|\widetilde{p}|}\right)^2}{m^2}\right)^2 . \tag{2.4}$$

The planar diagram, which is also present in the corresponding commutative-spacetime theory, diverges in the usual $\Lambda \to \infty$ limit. Instead the $\Lambda \to \infty$ limit of the nonplanar diagram is still finite as long as $\widetilde{p} \neq 0$. The divergence emerges only in the $\widetilde{p} \to 0$ (infrared) limit. Just like the UV portion of the loop integration introduces the Λ dependence of the planar diagram (2.2), it is the UV portion of the loop integration that introduces the dependence on $1/|\widetilde{p}|$, and the associated infrared singularity, of the nonplanar diagrams (2.3). This is a key aspect of the IR/UV mixing.

3. Effects of UV SUSY on IR physics

In this section we analyze a mass deformed Wess-Zumino model in canonical noncommutative spacetime. We emphasize the role that the UV scale of SUSY restoration plays in the IR sector of the model, and we also provide some more general remarks on the IR/UV mixing. This analysis will provide material for one of the points we raise in the later part of the paper, which concerns the nature of the bounds that can be set on the noncommutativity parameters using low-energy data.

3.1 A model with SUSY restoration in the UV

For definiteness, we present our observations, which have rather wide applicability, in the specific context of a mass deformed Wess-Zumino model, with action

$$S_{dwz} = S_0 + S_m + S_g , \tag{3.1}$$

$$S_0 = \int dx^4 \left\{ \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + \frac{1}{2}\overline{\psi}i\partial\!\!\!/\psi \right\} , \tag{3.2}$$

$$S_m = \int dx^4 \left\{ \frac{1}{2}F^2 + \frac{1}{2}G^2 + m_s F\varphi_1 + m_s G\varphi_2 - \frac{1}{2}m_f\overline{\psi}\psi \right\}, \tag{3.3}$$

$$\begin{aligned} S_g = \int dx^4 g \Big\{ & F \star \varphi_1 \star \varphi_1 - F \star \varphi_2 \star \varphi_2 + G \star \varphi_1 \star \varphi_2 + \\ & + G \star \varphi_2 \star \varphi_1 - \overline{\psi} \star \psi \star \varphi_1 - \overline{\psi} \star i\gamma^5 \psi \star \varphi_2 \Big\} . \end{aligned} \tag{3.4}$$

φ_1 and φ_2 are bosonic/scalar degrees of freedom, while ψ denotes fermionic spin-1/2 degrees of freedom. F and G are auxiliary fields. The model is exactly supersymmetric (SUSY) if $m_s = m_f$. We consider the case $m_s < m_f$ in which supersymmetry is only "restored" in the ultraviolet (UV), where both m_s and m_f are negligible with respect to the high momenta involved.

The free propagators are not modified by canonical noncommutativity:

$$\Delta_{m_s}(p) \equiv \Delta_{\varphi_1\varphi_1}(p) = \Delta_{\varphi_2\varphi_2}(p) = \frac{i}{p^2 - m_s^2 + i\varepsilon}, \qquad \Delta_{FF}(p) = \Delta_{GG}(p) = p^2\Delta_{\varphi_1\varphi_1}(p),$$

$$\Delta_{F\varphi_1}(p) = \Delta_{\varphi_1 F}(p) = \Delta_{\varphi_2 G}(p) = \Delta_{G\varphi_2}(p) = -m_s\Delta_{\varphi_1\varphi_1}(p), \qquad S(p) = \frac{i}{\not{p} - m_f}. \tag{3.5}$$

The vertices acquire the familiar θ-dependent phases:

$$V_{[\overline{\psi}\psi\varphi_1]} = -ig\cos(p_1\widetilde{p}_2), \qquad V_{[\overline{\psi}\psi\varphi_2]} = -i\gamma^5 g\cos(p_1\widetilde{p}_2),$$

$$V_{[F\varphi_1\varphi_1]} = ig\cos(p_1\widetilde{p}_2), \qquad V_{[F\varphi_2\varphi_1]} = -ig\cos(p_1\widetilde{p}_2), \qquad V_{[G\varphi_1\varphi_2]} = 2ig\cos(p_1\widetilde{p}_2). \tag{3.6}$$

(Notice that, taking into account momentum conservation at vertices, the momenta $p_1$and p_2 can be attributed equivalently to any of the three particles involved in each of the vertices.)

3.2 Self-energies and IR singularities

Self-energies will play a key role in our observations. Using the NC Feynman rules the self-energies for fermions and scalars can be evaluated straightforwardly. The one loop self-energy of the scalar field receives contributions from five Feynman diagrams, leading to the result

$$\begin{aligned} -i\Sigma_{1\text{loop}}(p) = -g^2 \int \frac{d^4k}{(2\pi)^4} \Big\{ & \left(8k^2 + 8m_s^2\right) \Delta_{m_s}(p)\Delta_{m_s}(p+k) + \\ & - \left(8k^2 + 8m_f^2 + 8p\cdot k\right) \Delta_{m_f}(p)\Delta_{m_f}(p+k) \Big\} \cos^2(k\widetilde{p}). \end{aligned} \tag{3.7}$$

This expression can be seen as the sum of three terms, and each of these terms is the sum of a planar and of a nonplanar part: $-i\Sigma_{1\text{loop}}(p) = I_1^P(p) + I_1^{NP}(p) + I_2^P(p) + I_2^{NP}(p) + I_3^P(p) + I_3^{NP}(p)$ with

$$\begin{aligned} I_1^P(p) + I_1^{NP}(p) \equiv & \frac{1}{2}g^2 \int \frac{dk^4}{(2\pi)^4} \frac{8k^2 + 8m_s^2}{(k^2 - m_s^2)\left((k+p)^2 - m_s^2\right)} + \\ & + \frac{1}{2}g^2 \int \frac{dk^4}{(2\pi)^4} \cos(2p\widetilde{k}) \frac{8k^2 + 8m_s^2}{(k^2 - m_s^2)\left((k+p)^2 - m_s^2\right)}; \end{aligned}$$

$$I_2^P(p) + I_2^{NP}(p) \equiv -\frac{1}{2}g^2 \int \frac{dk^4}{(2\pi)^4} \frac{8k^2 + 8m_f^2}{\left(k^2 - m_f^2\right)\left((k+p)^2 - m_f^2\right)} - $$
$$-\frac{1}{2}g^2 \int \frac{dk^4}{(2\pi)^4} \cos(2p\widetilde{k}) \frac{8k^2 + 8m_f^2}{\left(k^2 - m_f^2\right)\left((k+p)^2 - m_f^2\right)};$$
$$I_3^P(p) + I_3^{NP}(p) \equiv -\frac{1}{2}g^2 \int \frac{dk^4}{(2\pi)^4} \frac{8p\cdot k}{\left(k^2 - m_f^2\right)\left((k+p)^2 - m_f^2\right)} - $$
$$-\frac{1}{2}g^2 \int \frac{dk^4}{(2\pi)^4} \cos(2p\widetilde{k}) \frac{8p\cdot k}{\left(k^2 - m_f^2\right)\left((k+p)^2 - m_f^2\right)};$$

The planar terms involve integrations which are already done ordinarily in field theory in commutative spacetime. Their contributions lead, as in the commutative case, to logarithmic mass and wavefunction renormalization. We are here mainly interested in $\Sigma(p)^{NP(E)}_{1\text{loop}}$, the sum of the nonplanar contributions, which we study in the euclidean region. One easily finds[3]

$$\Sigma^{NP(E)}_{1\text{loop}}(p) = I_{1E}^{NP}(p) + I_{2E}^{NP}(p) + I_{3E}^{NP}(p)\,, \tag{3.8}$$

where

$$I_{1E}^{NP}(p) = \frac{g^2}{2\,(2\pi)^2} \int_0^1 da \Big\{ \Big[8m_s^2 + 4p^2(1-a)(2a-1)\Big] K_0(2|\widetilde{p}|\sqrt{m_s^2 + p^2 a(1-a)}\,) - $$
$$- \frac{4}{|\widetilde{p}|}\sqrt{m_s^2 + p^2 a(1-a)}\, K_1(2\,|\widetilde{p}|\,\sqrt{m_s^2 + p^2 a(1-a)}\,) \Big\}, \tag{3.9}$$
$$I_{2E}^{NP}(p) = -\left[I_{1E}^{NP}(p)\right]_{m_s \to m_f}, \tag{3.10}$$
$$I_{3E}^{NP}(p) = -\frac{4}{(2\pi)^2} p^2 \frac{g^2}{2} \int_0^1 db\, b K_0(2|\widetilde{p}|\sqrt{m_f^2 + p^2 b(1-b)})\,. \tag{3.11}$$

In the case of exact SUSY, $m_s = m_f$, the contributions I_{1E}^{NP} and I_{2E}^{NP} cancel each other, so that $\Sigma^{NP(E)}_{1\text{loop}} = I_{3E}^{NP}$ and there are no IR divergencies [10, 3].

In the general case, $m_s \neq m_f$, IR divergencies are present. Their structure depends on the relative magnitude of the SUSY-restoration scale $\Lambda_{SUSY} \simeq m_f$ and the noncommutativity scale $M_{nc} = 1/\sqrt{|\theta|}$ (where $|\theta|$ denotes generically a characteristic size of the elements of the matrix $\theta_{\mu\nu}$).

If $M_{nc} < m_f$ and $p \ll M_{nc}^2/m_f$ the non-planar part of the self energy is well approximated by

$$\Sigma^{NP(E)}_{1\text{loop}}(p) \simeq \frac{g^2}{(2\pi)^2} \int_0^1 da \Big\{ 6m_f^2 \ln\left(2\,|\widetilde{p}|\,\sqrt{m_f^2 + p^2 a(1-a)}\right) + $$
$$- 6m_s^2 \ln\left(2\,|\widetilde{p}|\,\sqrt{m_s^2 + p^2 a(1-a)}\right) + $$
$$+ 2p^2(1-a)(3a-1)\, \times$$
$$\times \left[\ln\left(\sqrt{m_f^2 + p^2 a(1-a)}\right) - \ln\left(\sqrt{m_s^2 + p^2 a(1-a)}\right)\right] +$$

[3] $K_0(x)$ and $K_1(x)$ are modified Bessel functions of the second kind.

$$+\left(m_s^2-m_f^2\right)[6\ln 2-6\gamma+1]+$$
$$+\,2p^2a\left[\ln\left(2\,|\widetilde{p}|\,\sqrt{m_f^2+p^2a(1-a)}\right)-(\ln 2-\gamma)\right]\Bigg\}. \quad (3.12)$$

[This approximation is also valid for all $p < M_{nc}$ if $M_{nc} > m_f$, but we are mainly interested here in the case $M_{nc} < m_f$ which allows us to explore the implications for low-energy phenomena of SUSY restoration above M_{nc}.]

If $M_{nc} < m_f$ and $M_{nc}^2/mf \ll p \ll M_{nc}$ the non-planar part of the self energy is well approximated by

$$\Sigma_{1\text{loop}}^{NP(E)}(p) \simeq \frac{g^2}{(2\pi)^2}\int_0^1 da\Bigg\{-\frac{1}{|\widetilde{p}|^2}-\ln\left(2\,|\widetilde{p}|\,\sqrt{m_f^2+p^2a(1-a)}\right)\times$$
$$\times\left[6m_s^2+2p^2(1-a)(3a-1)\right]+m_s^2[6\ln 2-6\gamma+1]+$$
$$+\,2p^2(1-a)\left[a\left(3\ln 2-3\gamma+\frac{1}{2}\right)-(\ln 2-\gamma)\right]\Bigg\}. \quad (3.13)$$

As a result of contributions coming from the UV portion of loop integrals, we are finding that (for $m_s \neq m_f$) the model is affected by logarythmic IR singularities (3.12) if $M_{nc}^2/mf \gg p$, but as soon as momenta are greater than M_{nc}^2/m_f the dependence of the self-energy on momentum turns into an inverse-square law (3.13). In the limit $m_f \to \infty$, the case in which there is absolutely no SUSY (not even in the UV), the inverse-square law takes over immediately and the theory is affected by quadratic IR singularities. The case of exact SUSY $m_f = m_s$ is free from IR singularities, but of no interest for physics (Nature clearly does not enjoy exact SUSY).

The IR/UV mixing manifests in two (obviously connected) ways which is worth distinguishing: (1) The UV portion of loop integrals is responsible for some IR singularities of the self-energies, (2) the low-energy structure of the model can depend on m_f even when m_f is much higher than the energy scales being probed. There is no IR/UV decoupling.

3.3 Further effects on the low-energy sector from UV physics

The implications of supersymmetry for the IR sector of canonical noncommutative spacetimes are very profound. In our illustrative model one finds that exact SUSY leads to absence of IR divergences, if SUSY is only present in the UV (UV restoration of SUSY) one finds soft, logarythmic, IR divergences, and total absence of SUSY ($m_f \to \infty$) leads to quadratic IR divergences. While the presence of SUSY in the UV is clearly an example of UV physics with particularly significant implications for the IR sector of canonical noncommutative spacetimes, from this example we must deduce that in general the loss of decoupling between UV and IR sectors can be very severe. Other features of the UV sector, which perhaps have not even yet been contemplated in the literature, might have similarly pervasive implications for the IR sector.

A particularly interesting scenario is the one in which supersymmetry is restored at some high scale (which in our illustrative model is m_f) and then at some even higher scale, possibly identified with the so-called "quantum-gravity scale", the theory predicts additional structures, which in turn, again, would affect the infrared. The example of quantum

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gravity is particularly significant since we have no robust (experimentally supported) information on this realm of physics, so it represents an example of UV physics for which our intuition migh easily fail, and as a consequence our intuition for its implications for the IR sector of a field theory in canonical noncommutative spacetime might also easily fail.

As a way to emphasize the sensitivity of the IR sector to such unknown UV physics, it is worth noting here some formulas that describe features of our illustrative model from the perspective of a theory with fixed cutoff scale Λ. For renormalizable field theories in commutative spacetime the presence of such a cutoff would be basically irrelevant: if the cutoff is much higher than all scales of interest it will negligibly affect all predictions and it can be uneventfully removed through the limit $\Lambda \to \infty$. Importantly, in a renormalizable field theory in commutative spacetime the limit $\Lambda \to \infty$ is uneventful independently of whether or not we have introduced in the theory all the correct UV degrees of freedom hosted by Nature: the low-energy physics is anyway independent of (decoupled from) the UV sector.

For field theories in canonical noncommutative spacetime the limit $\Lambda \to \infty$ is not at all trivial, meaning that the structures/degrees of freedom encountered along the limiting procedure can in principle affect significantly the low-energy physics. One can take the $\Lambda \to \infty$ limit in a physically meaningful way only under the assumption that one has complete knowledge of the full theory of Nature (something which of course we cannot even contemplate).

The sensitivity of the IR sector to unknown UV physics is well characterized by considering, for fixed cutoff scale Λ, the nonplanar contributions to the two point functions. For the two-point function we already considered previously one finds:

$$
\begin{aligned}
I_{1E}^{NP} &= \frac{g^2}{2}\left\{\frac{1}{(2\pi)^2}\int_0^1 da\,\left[8m_s^2 + 4p^2(2a-1)(1-a)\right] \times \right. \\
&\quad \times K_0\left(2\sqrt{\widetilde{p}^2+\frac{1}{\Lambda^2}}\sqrt{m_s^2+p^2a(1-a)}\right) + \frac{4}{\sqrt{\widetilde{p}^2+\frac{1}{\Lambda^2}}}\left[\frac{\widetilde{p}^2}{\widetilde{p}^2+\frac{1}{\Lambda^2}}-2\right]\times \\
&\quad \left. \times \sqrt{m_s^2+p^2a(1-a)}K_1\left(2\sqrt{\widetilde{p}^2+\frac{1}{\Lambda^2}}\sqrt{m_s^2+p^2a(1-a)}\right)\right\} \qquad (3.14)\\
I_{2E}^{NP} &= -I_{1E}^{NP}(m_s \to m_f) \qquad (3.15)\\
I_{3E}^{NP} &= -\frac{4}{(2\pi)^2}p^2\frac{g^2}{2}\int_0^1 dbbK_0\left(2\sqrt{\widetilde{p}^2+\frac{1}{\Lambda^2}}\sqrt{m_f^2+p^2b(1-b)}\right). \qquad (3.16)
\end{aligned}
$$

Note that nonplanar diagrams are cutoff by $\Lambda_{\text{eff}} = 1/\sqrt{\widetilde{p}^2+\frac{1}{\Lambda^2}}$. The self-energy is insensitive to the value of Λ as long as the condition $|\widetilde{p}| \gg 1/\Lambda$ is satisfied. But for $|\widetilde{p}| < 1/\Lambda$ there is an explicit dependence[4] on Λ signaling that the infrared sector is sensitive to new physics in the UV.

[4]It is worth noticing that for fixed cutoff Λ and $|\widetilde{p}| < 1/\Lambda$ the self-energy is essentially independent of the noncommutativity parameters. This is due to the fact that under those conditions the nonplanar contributions are completely negligible. This might encourage one to contemplate the possibility of a physical cutoff scale Λ, but it is important to notice that such a scale would be observer dependent since

4. Conditional bounds on noncommutativity parameters from low-energy data

The main point of our manuscript is that the observations made in the previous section have significant implications for the comparison of low-energy experimental data with a theory in canonical noncommutative spacetime.

It is useful to note here a brief description of the conventional technique that allows to use low-energy data to set absolute (unconditional!) limits on the parameters of theories in commutative spacetime:

- **1C.** Data are taken in experiments involving particles with energies/momenta from some lower (IR) limit, $\mathcal{S}_{\min}$ (we of course do not have available probes with wavelength, e.g., larger than the size of the Universe) up to an upper limit, $\mathcal{S}_{\max}$, which naturally coincides with the highest energy scales attainable in our laboratory experiments (and, in appropriate cases, the energy scales involved in certain observations in astrophysics).

- **2C.** We then compare these experimental results obtained at energy/momentum scales within the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ to the corresponding predictions of the theory of interest. In deriving these predictions we sometimes formally appear to use the whole structure of the theory, all the way to infinite energy/momentum; however, in reality, because of the IR/UV decoupling that holds in (renormalizable) theories in commutative Minkowski spacetime, the theoretical prediction only depends on the IR structure of the theory, up to energy/momentum scales which are not much bigger than $\mathcal{S}_{\max}$. (For example, degrees of freedom with masses of order, say, $10^5\mathcal{S}_{\max}$ would anyway not affect the relevant predictions).

- **3C.** If the theoretical predictions obtained in this way do not agree with the observations performed in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ we then conclude that the theory in question is to be abandoned.

- **4C.** If the theoretical predictions obtained in this way agree with the observations performed in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ we then conclude that the theory in question provides a valid description of phenomena up to energy/momentum scales of order $\mathcal{S}_{\max}$. Typically the predictions of the theory will depend on some free parameters and this parameter space will be constrained by the requirement of agreeing with the observations. Values of the parameters that do not belong to this allowed portion of the parameter space are definitely (unconditionally) excluded, since nothing that we could introduce in the ultraviolet could modify the low-energy predictions. In

ordinary Lorentz transformations still govern the transformations between inertial observers in canonical noncommutative spacetime [18]. (In other noncommutative spacetimes, where the action of boosts is deformed, a cutoff scale can be introduced in an observer-independent way [5, 18], but this is not the case of canonical noncommutative spacetimes.) We shall disregard this possibility; however, in theories that already identify a preferred class of inertial observers, such as theories in canonical noncommutative spacetimes, the possibility of an observer-dependent cutoff scale cannot [18] be automatically dismissed.

light of the fact that the structure of the theory above $\mathcal{S}_{\max}$ did not play any true role in the derivation of the predictions, the successful comparison with $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ experiments provides no particular encouragement for what concerns the validity of the theory at scales much above $\mathcal{S}_{\max}$.

- **5C.** With precision measurements in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ we can sometimes put limits on features of the theory also slightly (up to a few orders of magnitude) above $\mathcal{S}_{\max}$. For example, one of the parameters of the theory could be the mass of a certain particle and the contributions to low-energy processes due to that particle, while suppressed by its mass, can be tested in high-precision measurements.

For theories in canonical noncommutative spacetime the situation is quite different, as one infers from the analysis reported in the previous section. The comparison between the theory and data taken in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ is much more delicate:

- **2NC.** From the observations made in the previous section it follows that in a canonical noncommutative spacetime a truly reliable derivation of the predictions for the energy/momentum range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ requires full knowledge of the theory at all energy/momentum scales up to $M_{nc}^2/\mathcal{S}_{\min}$ (and of course, if $M_{nc} \gg \mathcal{S}_{\max}$, the scale $M_{nc}^2/\mathcal{S}_{\min}$ can be much higher than both M_{nc} and $\mathcal{S}_{\max}$). In particular, the IR/UV mixing is such that degrees of freedom with masses that are much above $\mathcal{S}_{\max}$ still affect significantly the predictions of the theory in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$.

- **3NC.** So the theory can only be taken as a full description of Nature. It cannot be intended to give the right predictions only in some low-energy limit. If the predictions of such a theory are found to be in conflict with observations, it might still well be that the theory contains the right low-energy degrees of freedom, and that the disagreement is due to having adopted the wrong UV sector. So, from our more conventional perspective (in which we try to identify theories that contain the right degrees of freedom up to a certain scale) disagreement with observations does not force us to abandon the theory: it only invites us to introduce appropriate new physics in the UV sector.

- **4NC.** Similarly, if the theoretical predictions are found to agree with the observations performed in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ when some free parameters fall within a certain allowed portion of parameter space, values of the parameters that do not belong to that region of the parameter space cannot be conclusively excluded. They are excluded only **conditionally**, in the sense that their exclusion is only tentative, pending further exploration of the UV sector. Think for example of the illustrative model we considered in the preceding section. The $m_f \to \infty$ of that model is a model without any SUSY (not even in the UV sector). One could propose such a non-SUSY model and compare it to data obtained in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$. Clearly the need to agree with observations would then impose a severe (lower) bound on the noncommutativity scale, a key parameter of the theory, in order to suppress the IR divergences (e.g. effectively relegating those divergences at scales below $\mathcal{S}_{\min}$).

However, this bound on the noncommutativity scale would be only conditional, in the sense that modifying the theory only in the ultraviolet (i.e. where we would say it has not been tested with our data in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$) may be sufficient to lift the bound. In fact, SUSY in the ultraviolet sector (m_f large but finite) significantly softens the divergences used to set the bound. Whereas in commutative spacetime the bounds on parameter space apply directly to the structure of the theory in the range of energy/momentum scales that have been probed experimentally, in canonical noncommutative spacetime the information gained experimentally in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ leaves open two possibilities: it may still, as in the case of theories in commutative spacetime, constrain the parameters of the theory in that same range of energy/momentum scales, but one cannot exclude the possibility that our low-energy observations are instead primarily a manifestation of some features of the UV sector (transferred to the low-energy sector via the IR/UV mixing) and therefore cannot be used to constrain the low-energy structure of the theory. If there is disagreement between theory and experiments in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ one would normally assume that some aspects (*e.g.* the field content) of the theory must be changed in that same range of energy/momentum scales, instead in canonical noncommutative spacetime that same disagreement could be solved not only by introducing new features in the $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ region but also by introducing new features in the UV sector of the theory.

- **5NC.** Since data taken in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ do not even give definitive information on the structure of the theory in that same range, it is of course true that measurements in the range $\{\mathcal{S}_{\min}, \mathcal{S}_{\max}\}$ cannot be used to put limits on features of the theory even just slightly above $\mathcal{S}_{\max}$, no matter how precise those measurements are. However, just because features of the UV sector affect the low-energy physics, under the assumption that the spacetime is indeed canonically noncommutative, one can gain insight of the UV structure of the theory, even just using low-energy data. For example, some of the observations made in the previous section provide an opportunity to discover UV SUSY even just using low-energy data: if data allowed us to identify an energy/momentum scale at which the self-energy changed its qualitative dependence on momentum in the way described by comparison of eqs. (3.12) and (3.13), we could then infer rather robustly the presence of SUSY at high energies and (if the value of the noncommutativity scale was deduced from some other observations) we could even deduce the scale of SUSY restoration.

5. Futility of some approaches based on expansion in powers of θ

The observations reported in the preceding section indicate that some of the standard techiques used in phenomenology require a prudent implementation in the context of theories in canonical noncommutative spacetimes. We want to emphasize in this section that for one of the techniques which served us well in the analysis of theories in commutative spacetime there are even more severe limitations to the applicability in the context of theories in canonical noncommutative spacetimes. This is the technique that relies on the truncation of a power series in one of the parameters of the theory: we argue that, at the quantum-field-theory level, the results obtained by truncating a power series in θ do not provide a reliable approximation of the full theory. This type of truncation, which has been widely used in the literature [19]–[25]), is based on the inclusion of only a few terms in the θ-expansion of the Moyal $\star$-product. For example up to the second order in θ one could write

$$\varphi_1(x)\star\varphi_2(x)=\varphi_1(x)\varphi_2(x)+\frac{i}{2}\theta^{\mu\nu}\partial_\mu\varphi_1(x)\partial_\nu\varphi_2(x)-\frac{1}{8}\theta^{\alpha\beta}\theta^{\mu\nu}\partial_\alpha\partial_\mu\varphi_1(x)\partial_\beta\partial_\nu\varphi_2(x)+O(\theta^3)\,. \tag{5.1}$$

The resulting action constructed with the truncated $\star$-product (5.1) depends only on a finite number of derivatives so it is local, unlike the full theory. Moreover, since θ has negative mass dimensions, the action will also certainly be power-counting nonrenormalizable, whereas the full theory might be renormalizable [2, 10, 26, 27, 28].

Even more serious concerns emerge from the realization that the expansion one is performing is (of course) not truly based on a power series in the dimensionful quantity θ: it is rather an expansion in dimensionless quantities of the type $p\theta p$. Therefore already at tree level the truncated θ-expanded theory can only give a good approximation of the full theory at scales p such that $p\theta p \lesssim 1$, i.e. $p \lesssim 1/\sqrt{\theta}$.

But if $\Lambda > 1/\sqrt{\theta}$ (i.e., if we accept our ignorance of the correct theory at least at scales above a certain Λ, higher than $1/\sqrt{\theta}$) even for $p \lesssim 1/\sqrt{\theta}$ the expansion cannot be used reliably. Its reliability is spoiled by quantum corrections. The quantum corrections involve the Moyal $\star$-product inserted in loop diagrams, and the truncation will reliably describe these loop corrections only for loop momenta such that $p \lesssim 1/(\theta\Lambda)$. In fact, in loop integrals involving factors of the type $p\theta k$, with p playing the role of external momentum and k playing the role of integration/loop momentum, one would like a reliable truncation that is valid over the whole loop-integration range, which extends at least up to a cutoff Λ. In order to have $p\theta k \lesssim 1$ even for k as large as Λ it is necessary to assume that indeed $p \lesssim 1/(\theta\Lambda)$. This can also be inferred straightforwardly in the illustrative example of the "$\lambda\Phi^4$" scalar-boson field theory: there one finds that the full theory predicts nonplanar terms giving a leading contribution of the form

$$\Sigma^1_{NP}(p) \simeq \frac{g^2}{\widetilde{p}^2+1/\Lambda^2} = \Lambda^2\frac{g^2}{\Lambda^2\widetilde{p}^2+1}\,. \tag{5.2}$$

whereas the truncated θ-expansion of the $\star$-product would replace this prediction with

$$\Sigma^1_{NP}(p) \simeq g^2\Lambda^2\left\{1-\Lambda^2\widetilde{p}^2+O(\theta^4)\right\}. \tag{5.3}$$

Clearly the two expressions are equivalent only if $\Lambda^2\widetilde{p}^2 \lesssim 1$, which indeed corresponds to $p \lesssim 1/(\theta\Lambda)$.

Therefore, when one includes quantum/loop effects, the truncated θ-expansion could be a good approximation of the full theory only in the range of momenta $p \lesssim 1/(\theta\Lambda)$. But as we have discussed in the preceding section this is just the range of momenta in which the theory is maximally sensitive to ultraviolet physics, which we must assume to be unknown. In other words the truncated θ-expansion reliably approximates the full theory only in a

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regime where the full theory is itself void of predictive power, because of its sensitivity to unknown physics that might be present in the ultraviolet. It therefore appears that these truncated θ-expansions cannot be used for a meaningful comparison between data and theories in canonical noncommutative spacetime. In other contexts expansions in powers of p versus some characteristic momentum scale have been proven to give a reliable low-energy effective-theory description of the full theory one intends to study, but in this case of field theories in canonical noncommutative spacetime the IR/UV mixing provides a powerful obstruction for any attemp to obtain a meaningful low-energy effective-theory description.

6. Implications for other previous θ-limit analyses

A key point that emerged from the analysis reported in the previous sections is that a conventional phenomenological study, focusing on low-energy data, can only provide conditional bounds on the noncommutative parameters θ, conditional to some assumption on the ultraviolet sector (which can modify the low-energy physics through IR/UV mixing). In this section we argue that some previous studies, although not emphasizing the point we stressed here, actually did encounter related issues, which were a manifestation of the challenge that has been here of concern. Our discussion will focus on the representative studies reported in refs. [14, 30, 31].

In ref. [14] bounds on the noncommutative parameters θ have been obtained from the analysis of noncommutative Yukawa theories with lagrangian of the type

$$L = i\overline{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial\phi)^2 + g\overline{\psi}\star\phi\star\psi - m\overline{\psi}\psi\,. \tag{6.1}$$

In this theories, under the assumption that $\theta\Lambda^2 \ll 1$, quantum fluctuations generate, at the two-loop level, terms of the type

$$m\Lambda^2\left(\frac{g^2}{16\pi^2}\right)^2\theta^{\mu\nu}\overline{\psi}\sigma_{\mu\nu}\psi\,, \tag{6.2}$$

where Λ is the familiar hard-momentum cutoff. Terms of this type are strongly constrained from various experimental measurements, leading [14] to strong bounds on $\theta\Lambda^2$. However, as the authors of ref. [14] noticed, these bounds cannot be immediately translated into bounds on θ because of the dependence on the cutoff: the bounds are of the type $\theta\Lambda^2 < [\theta\Lambda^2]_{\max}$ rather than of the type $\theta < [\theta]_{\max}$. A different assumption for the cutoff Λ leads to a different bound on θ.

Moreover, whereas our familiar bounds on theories in commutative spacetimes are completely insensitive to the ultraviolet physics, the bounds obtained in ref. [14] also have some nonnegligible dependence on the ultraviolet physics, i.e. the degrees of freedom above the cutoff scale Λ. This was not observed in ref. [14] but it can be easily verified by comparing the results of ref. [14] and the results of ref. [30]. In fact, in ref. [30] there is a scale of SUSY restoration, which again separates a low-energy regime and a high-energy regime. Assuming the same value of the (SUSY-restoration/cutoff) scale Λ, the theories of ref. [14] and ref. [30] are identical up to the scale Λ. Above Λ the theory of ref. [14] is simply cutoff (no degrees of freedom) while the theory of ref. [30] is supersymmetric. And the fact that the two theories are different in the ultraviolet is sufficient for them to give raise to different "low-energy bounds": ref. [30] also obtains bounds of the type $\theta\Lambda^2 < [\theta\Lambda^2]_{\max}$ but with some nonegligible quantitative differences with respect to the corresponding results of ref. [14]. Those quantitative differences are a manifestation of the sensitivity to the (in-principle unknown) ultraviolet sector.

Our concern becomes more severe in cases in which a dependence of the bounds on an ultraviolet scale if not even acknowledged. It is not uncommon in the literature on these theories in canonical noncommutative spacetime to encounter experimental bounds announced without a warning about their sensitivity (and conditionality) to the ultraviolet. For example, the otherwise elegant phenomenological analysis reported in ref. [31] leads to results which the authors describe as absolute limits on the noncommutativity parameters θ (no mention of a dependence on the ultraviolet physics) which of course cannot be correct. The careful reader will easily verify that actually the results obtained in ref. [31] depend crucially on the assumption of a relatively low (still "ultraviolet") quantum-gravity scale, not much above the TeV scale. In a more general setting for theories in canonical noncommutative spacetime the limits obtained in ref. [31] could be circumvented by appropriate introduction of UV features, since they are based on low-energy data.

7. Conclusions

Clearly the type of IR/UV mixing which is present in field theories in canonical noncommutative spacetime has wide implications for the strategies that should be adopted in order to falsify/verify these theories. Theories that (according to our conventional language) differ only in an experimentally unaccessible range of momenta may give rise to different predictions in the low-energy regime. The bounds on parameter space that one usually is able to set using low-energy data are here only conditional, in the sense clarified in section 4. On the other hand low-energy data can be used to gain insight on the UV sector, as we discussed for the specific case of UV SUSY, under the assumption that the theory does indeed live in a canonical noncommutative spacetime.

The implications of the IR/UV mixing are clearly very severe for self-energies. In the models so far studied it appears [2, 3, 10, 29] that instead the implications for interaction vertices are less significant. This might mean that tests based on the properties of interaction vertices could be more indicative (less conditioned on assumptions concerning the UV sector) than tests based on properties of the self-energies. However, it is perhaps best to be cautious in formulating this expectation in general terms: in these theories we are not protected by the usual IR/UV decoupling and the fact that in certain specific models one finds that interaction vertices are only moderately sensitive to properties of the UV sector cannot provide a general reassurance.

Some of the points we raised here clarify that in the investigation of theories in canonical noncommutative spacetime it might be necesssary to "build a case" in favour or against consistency with observations (whereas in commutative spacetime a single experiment can

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give conclusive unconditional indications). The case would be built by considering a variety of data, and observing that they are all consistent with the characteristic structure of theories in canonical noncommutative spacetime. Because of the nature of these characteristic features of canonical noncommutativity, it can be very useful to rely on data that concern a wide range of energy scales. The astrophysical studies analyzed, for what concerns canonical noncommutativity, in ref. [16] could play an important role in this programme. For example, the larger range of energies explored, if astrophysical data on particle propagation were combined with the corresponding laboratory data, could allow to establish that at some particular momentum scale the dependence of the self-energy on momentum suffers a transition of the type described in eqs. (3.12) and (3.13), which is a characteristic feature of theories in canonical noncommutative spacetime with (softly broken) UV SUSY.

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